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A FAMILY OF EXACT SOLUTIONS OF THE EQUATIONS OF THE ONE-DIMENSIONAL MOTION OF A GAS UNDER THE INFLUENCE OF MONOCHROMATIC RADIATION*

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A new family of exact solutions with a linear dependence of the velocity on the spatial coordinate is constructed for the equations of the non-stationary motion of an ideal gas taking account of the absorption of monochromatic radiation. The solutions contain some arbitrary functions and some arbitrary constants. Exact solutions without radiation are known /1, 2/. Taking account of the absorption of monochromatic radiation, the problem was developed in /3/, and exact selfsimilar solutions were found. Unlike these solutions, we construct below solutions that contain some arbitrary functions.

The one-dimensional motion of an ideal gas, taking account of the absorption of monochromatic radiation, is described by the system of equations

$$u_t + uu_r + \rho^{-1}p_r = 0, \quad \rho_t + u\rho_r + \rho\,(u_r + \nu r^{-1}u) = 0$$
$$p_t + up_r + \gamma p\,(u_r + \nu r^{-1}u) = (\gamma - 1)\,kj, \quad j_r + \nu r^{-1}j = kj \tag{1}$$

where ρ is the density, p is the pressure, u is the velocity, γ is the constant ratio of the specific heats, j is the intensity of the counter-radiation (the flow of radiant energy through a unit area in unit time) and k is the absorption coefficient, $\nu = 0, 1, 2$ for plane, cylindrical and spherical symmetry, respectively. Here, we do not take account of the thermal conductivity or viscosity of the gas, the radiation of the medium or the scattering of radiant energy (see /3, 4/).

Let $G(z)$, $f(t)$ be arbitrary functions of their arguments, and let c be an arbitrary constant. We can ascertain that the formula

$$\mu\mu^{\cdot\cdot} - 2(\mu^{\cdot})^2 - \mu^{\gamma+3+\nu(\gamma-1)}\mu_2 = 0 \tag{2}$$
$$\mu\mu_1^{\cdot\cdot} - 2\mu^{\cdot}\mu_1^{\cdot} - \mu^{\gamma+2}\mu_1\mu_2 = 0$$
$$G_{\nu1}(z) = \frac{dG}{dz} + \frac{\nu G}{z}, \quad G_{\nu2}(z) = \frac{dG_{\nu1}}{dz} \tag{3}$$
$$z = r\mu - \mu_1, \quad u = -\mu^{-1}r\mu^{\cdot} + \mu^{-1}\mu_1$$
$$p = \mu^{\gamma(1+\nu)}\mu_2 G_{\nu1}(z) + c\mu^{\gamma(1+\nu)}, \quad \rho + z^{-1}\mu^{1+\nu}G_{\nu2}(z)$$
$$j = (\gamma-1)^{-1}\mu^{(1+\nu)\gamma-1}\,[G(z) + r^{-\nu}f(t)]\,\mu_2^{\cdot}, \quad k = \frac{\mu G_{\nu1}(z)}{G(z) + r^{-\nu}f(t)}$$

(a dot indicates differentiation with respect to time t) with the additional condition $\mu_1 = 0$ for $\nu = 1, 2$, defines an exact solution of Eq.(1).

If we set $\mu_2 = \mathrm{const}$, we obtain a class of exact solutions of the radiationless equations of gas dynamics.

We will give some example.

1°. If we set $\mu_1 = 0$, $\mu_2 = \mathrm{const}$, we obtain Sedov's family of exact solutions of the equations of gas dynamics /1/, which has been applied in a number of problems /2, 4/.

2°. Setting

$$\mu = t^{-\lambda}, \quad \mu_1 = \frac{D}{1-\gamma}\,t^{1-\lambda}, \quad \mu_2 = 2\,\frac{\gamma-1}{(\gamma+1)^2}$$
$$\lambda = \frac{2}{\gamma+1}, \quad G_{01} = \frac{b\,(\gamma-1)}{2\gamma}\,z^{2+\varkappa}, \quad \varkappa = \frac{2}{\gamma-1}$$

in the plane case, we have the exact solutions of equations of hydrodynamics (1), without radiation that are used in the theory of detonations (D is the wave velocity and b, c are arbitrary constants)

$$u = \frac{2}{\gamma+1}\left(\frac{r}{t} - \frac{D}{2}\right), \quad p = \frac{b}{\gamma}\left(\frac{\gamma-1}{\gamma+1}\right)^2\left(\frac{r}{t} + \frac{D}{\gamma-1}\right)^{2+\varkappa} + ct^{-\lambda\gamma}$$
$$\rho = b\left(\frac{r}{t} + \frac{D}{\gamma-1}\right)^{\varkappa}, \quad j = 0.$$

3°. Let $\mu > 0$ be an arbitrary function of t in the plane case, $\mu_1 = f = 0$, $\mu_2 = \mu^{-\gamma-3}\,[\mu\mu^{\cdot\cdot} - 2(\mu^{\cdot})^2]$, $G(z) = z^{\alpha}$, α is a number, $\alpha(\alpha-1) \neq 0$. Then Eqs.(2) are obeyed and according to formula

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(3) we have

$$u = r\mu^{-1}\mu', \quad p = \alpha\mu^{\gamma}\mu_z z^{\alpha-1} + c\mu^{\gamma}$$
$$\rho = \alpha(\alpha-1)\mu z^{\alpha-3}, \quad j = (\gamma-1)^{-1}\mu^{\gamma-1}z^{\alpha}\mu_z'$$
$$z = \mu r, \quad k = \alpha r^{-1}.$$

The suggestion to look for exact solutions of (1) with a linear dependence of the velocity on the spatial coordinate was made by V.P. Korobeinikov.

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THE INFLUENCE TENSOR FOR AN ELASTIC MEDIUM WITH POISSON'S RATIO VARYING IN ONE DIRECTION*

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We construct an analogue of the known Kelvin-Somigliana tensor for an unbounded elastic medium with a Poisson's ratio that varies in one direction and a constant shear modulus. We deduce the corresponding force tensor. We also consider the effect of the temperature. The effect of inhomogeneity is demonstrated by examples.

1. Initial relations. We can attach the following form to the resolving equations of the linear theory of elasticity of the inhomogeneous medium under consideration in a Cartesian coordinate system x_i $(i = 1, 2, 3)$ /2/:

$$\Delta m = -\Phi_{1,1} - \Phi_{2,2} + \Phi_{3,3}, \quad \Delta n = -\Phi_{1,2} + \Phi_{2,1} \tag{1.1}$$

$$\Delta k = (1-\nu)^{-1}[m_{,33} + \nu(\Phi_{1,1} + \Phi_{2,2}) + (1+\nu)\alpha\theta] - \Phi_{3,3} \tag{1.2}$$
$$(X_i = 2\mu\gamma^2\Phi_i \ (i = 1, 2), \ X_3 = 2\mu\Phi_{3,33}).$$

Here m, k, n are resolving potential functions, X_i are components of the volume force vector, Φ_i are arbitrary volume force potential functions, θ is the temperature, α is the coefficient of linear expansion, ν is Poisson's ratio, μ is the shear modulus, Δ is the Laplace operator and γ^2 is the two-dimensional Laplace operator (in the variables x_1 and x_2). Partial derivatives are indicated by a comma followed by the index of the corresponding variable.

The components of the dislocation vector u_i and the stress tensor σ_{ij} are expressed in terms of the potential functions according to the formulae

$$u_i = (k+m)_{,i} + 2(\varepsilon_{ip3}n_{,p} - \delta_{3i}m_{,3}) \tag{1.3}$$
$$\sigma_{ij} = 2\mu\{(k+m)_{,ij} + \varepsilon_{jp3}n_{,ip} + \varepsilon_{ip3}n_{,pj} - \delta_{3i}m_{,3j} - \delta_{3j}m_{,3i} +$$
$$(1-2\nu)^{-1}[\nu\Delta(k+m) - 2\nu m_{,33} - (1+\nu)\alpha\theta]\}$$

(δ_{ij} is the Kronecker delta and ε_{ijp} are components of the Levi-Civita tensor).

In the relations we have noted, Poisson's ratio is everywhere taken to be an arbitrarily differentiable or, in the general case, partially-differentiable function of one variable x_3,

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